# Rossmoyne Senior High School

### Year 11 Examination, 2014

### Question/Answer Booklet

**SOLUTIONS**

# MATHEMATICS: SPECIALIST 3A/3B

## Section Two:

## Calculator-assumed

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Student Number: In figures |  |  |  |  |  |  |  |  |

In words

Your name

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time for this section: one hundred minutes

## Materials required/recommended for this section

##### *To be provided by the supervisor*

This Question/Answer Booklet

Formula Sheet (retained from Section One)

##### *To be provided by the candidate*

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination.

## Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
| Section One:  Calculator-free | 7 | 7 | 50 | 50 | 33⅓ |
| Section Two:  Calculator-assumed | 12 | 12 | 100 | 100 | 66⅔ |
|  | | | **Total** | 150 | 100 |

## Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

* Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
* Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

1. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
2. It is recommended that you **do not use pencil**, except in diagrams.

Section Two: Calculator-assumed (100 Marks)

This section has**twelve (****12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 (8 marks)

The latitude and longitude of three cities are given in this table, to the nearest degree.

|  |  |  |
| --- | --- | --- |
| *City* | *Latitude* | *Longitude* |
| Jiujiang (China) | 30°N | 116°E |
| Perth (Australia) | 32°S | 116°E |
| Austin (USA) | 30°N | 98°W |

Assume the radius of the earth is 6350 km.

(a) Calculate the distance between Perth and Jiujiang along their common line of longitude.

(2 marks)



(b) Calculate the distance between Jiujiang and Austin along their common line of latitude.

(3 marks)



(c) The town of Forster, on the east coast of Australia, is 3570 km away from Perth and on the same line of latitude. Determine the longitude of Forster, to the nearest degree.

(3 marks)



Question 9 (7 marks)

Two vectors are given by  and .

(a) Determine the value(s) of  if  and  are

(i) parallel. (2 marks)



(ii) perpendicular. (3 marks)



(b) If , determine the angle between  and  to the nearest degree. (2 marks)



Question 10 (7 marks)

A small radio controlled boat leaves point A on a river bank and heads off at a constant speed on a bearing of 120°. The operator is standing 100 metres due east of point A and notes that after 45 seconds, the boat is 55 m away from her position.

(a) Use the above information to complete the diagram below. (1 mark)



(b) Calculate the two possible distances travelled by the boat in the 45 seconds. (4 marks)



At this time, after 45 seconds, the operator turns the boat so that it heads directly towards their position without changing its speed.

(c) Determine the minimum possible time that the boat will take to reach the operator.

(2 marks)

Maximum speed is .

Minimum time is .

Question 11 (9 marks)

(2, 3), (1, -2) and (-3, 1) are the vertices of a triangle.

(a) State the vector . (1 mark)



(b) Determine the exact value of . (2 marks)



(c) Determine the vector equation of the line

(i) through  parallel to . (2 marks)



(ii) through  perpendicular to . (2 marks)



(d) A circle with centre at C passes through (0, 0). Determine the vector equation of this circle.

(2 marks)



Question 12 (7 marks)

Point  has polar coordinates  and point  has Cartesian coordinates .

(a) Convert the polar coordinates of point  into exact Cartesian coordinates.

(1 mark)



(b) Convert the Cartesian coordinates of point  into polar coordinates , where  and . (1 mark)



(c) Plot the points  and  on the axes below. (2 marks)



(d) If  is the origin, determine

(i) the size of . (1 mark)



(ii) the length . (1 mark)



(iii) the area of the triangle . (1 mark)



Question 13 (10 marks)

The area of an oil slick around a ship,  in square metres,  minutes after the rupture of the boats fuel tank, was modelled by



30 minutes after the tank was ruptured, the crew of the ship took steps to stem the fuel leakage.

(a) Determine the area of the oil slick after

(i) 30 minutes. (1 mark)

391 m2

(ii) 60 minutes. (1 mark)

639 m2

(b) Sketch the graph of  on the axes below. (3 marks)



(c) Determine the time taken for the area of the slick to treble in size from 200 m2 to 600 m2, giving your answer in minutes to one decimal place. (3 marks)



(d) Comment on how the size of the oil slick is changing several hours after the initial rupture.

(2 marks)

The slick is increasing at a decreasing rate.

As  increases, the size of the slick is tending to 650 m2, because .

Question 14 (11 marks)

Consider the function .

(a) Draw the graph of  on the axes below. (3 marks)



(b) Write a piecewise definition of . (3 marks)



Let , where .

(c) For which values of  is  constant? (2 marks)



(d) Write a piecewise definition of . (3 marks)



Question 15 (6 marks)

A function is defined as .

Determine the constants  and  if  is continuous and differentiable everywhere.



Question 16 (10 marks)

The graphs of  and  are shown below over their respective domains.



(a) Determine

(i) . (1 mark)



(ii) . (2 marks)



(b) Determine

(i) the range of . (1 mark)



(ii) the domain for which  is defined. (2 marks)

Range of  must be restricted to be within domain of :



(c) The defining rule for .

Determine the values of . (4 marks)



Question 17 (8 marks)

 is a parallelogram with  and .



 is the point on side  such that .

(a) Express in terms of  and :

(i) . (1 mark)



(ii) . (1 mark)



(iii) . (1 mark)



 is the point on  such that ,  and  are collinear.

(b) If  and , use the fact that  to determine the values of  and . (5 marks)



Question 18 (7 marks)

Consider the number patterns below.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 10 |  |  | 0.665 |
| 100 |  |  | 0.961 |
| 1000 |  |  | 0.996 |

(a) Calculate , rounding your answer to four decimal places. (1 mark)

0.1353

(b) Write a formula for  and for  in terms of , where  is a positive integer.

(2 marks)



(c) Determine the values of ,  and . (2 marks)



(d) Determine the exact limiting values of ,  and  as . (2 marks)



Question 19 (10 marks)

At noon, a jet fighter flying at a constant altitude and at position  km, is given instructions to refuel in mid-air from a tanker aircraft flying at the same altitude and at position  km.

The jet fighter is told to fly at a constant velocity of 1 150 km/h in order to intercept the tanker aircraft, which is flying with a constant velocity given by  km/h.

Suppose the velocity vector the jet fighter needs to maintain for interception is  km/h.

(a) Explain why . (1 mark)

By considering the speed of the jet,



(b) Determine a vector  for the initial position of the jet relative to the tanker. (1 mark)



(c) Determine a vector  for the velocity of the jet relative to the tanker. (1 mark)



(d) State a relationship between  and  that must hold if the jet is to intercept the tanker after  hours. (1 mark)



(e) Determine the values of  and . (4 marks)



Solve simultaneously to get



, so discard this solution set.

Hence .

(f) Calculate the position vector of the jet at the instant it intercepts the tanker, giving coefficients to the nearest km. (2 marks)



Additional working space

Question number: \_\_\_\_\_\_\_\_\_

Additional working space

Question number: \_\_\_\_\_\_\_\_\_

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